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Keeping a Spacecraft on the Sun-Earth Line

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Overview

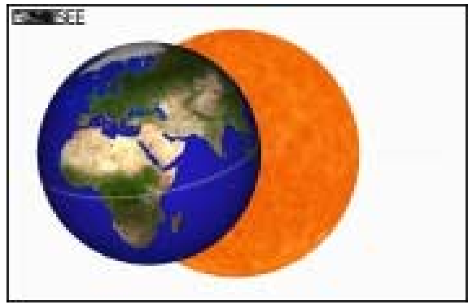
- The need to keep a S/C on the Sun-Earth line
- Negating the lunar perturbation with propulsion
- Approximation of the lunar perturbation
- Estimate of monthly ΔV requirement
  - S/C fixed at Sun-Earth $\mathcal{L}_2$
  - S/C free to move along Sun-Earth line
- Excursions with and without lunar perturbation
- Conclusion

Why keep a S/C on the Sun-Earth Line?

- Study Earth's atmosphere as it occults sunlight
  - Hourly measurements at all latitudes
  - Global, high-resolution 3D maps of CO_2, O_3, O_2, CH_4, H_2O, N_2O
  - Can't be done continuously or globally from LEO
- Sun-Earth $\mathcal{L}_2$ offers a unique vantage point
  - Must stay within 200 km of the Sun-Earth line
- "Standard" orbits won't work
  - Lissajous and halo orbits stray far from Sun-Earth line
  - Nearly rectilinear halo orbits are perpendicular to line between primaries, and don't account for 4th body perturbation

Views from the Neighborhood of Sun-Earth $\mathcal{L}_2$

0.65°



$X = X^*, \ Y = 5{,}000$ km

1.0°

$X^* = 1.5082 \times 10^6$ km

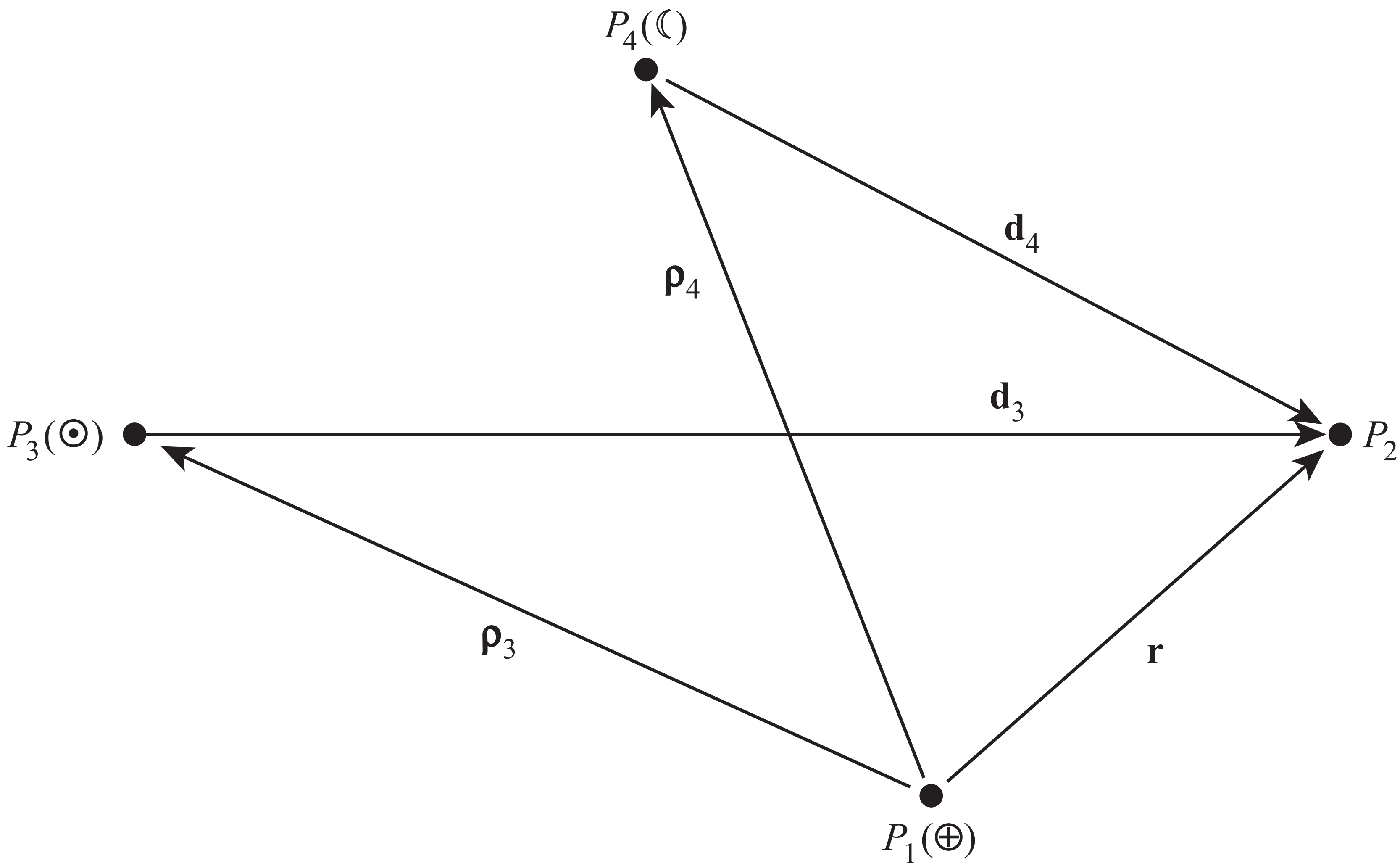
$X = X^*, \ Y = 200$ km

$X = X^* - 50{,}000$ km

$X = X^*, Y = 0$

$X = X^* + 50{,}000$ km

Four-body System



Relative Motion of Earth and Spacecraft

Motion of Earth & S/C perturbed by Sun and moon

$$\frac{{}^N d^2\mathbf{r}}{dt^2} + \frac{G(m_1+m_2)\mathbf{r}}{r^3} = \frac{\mathbf{p}}{m_2} - G\left[m_3\left(\frac{\mathbf{d}_3}{d_3^{\,3}} + \frac{\vec{\rho}_3}{\rho_3^{\,3}}\right) + m_4\left(\frac{\mathbf{d}_4}{d_4^{\,3}} + \frac{\vec{\rho}_4}{\rho_4^{\,3}}\right)\right]$$

where $\mathbf{p}/m_2$ is propulsive force per unit mass applied to S/C.
Choose $\mathbf{p}/m_2$ to cancel lunar perturbation,

$$\frac{\mathbf{p}}{m_2} = Gm_4\left(\frac{\mathbf{d}_4}{d_4^{\,3}} + \frac{\vec{\rho}_4}{\rho_4^{\,3}}\right)$$

to reduce the four-body problem to a three-body problem:

$$\frac{{}^N d^2\mathbf{r}}{dt^2} + \frac{G(m_1+m_2)\mathbf{r}}{r^3} = -Gm_3\left(\frac{\mathbf{d}_3}{d_3^{\,3}} + \frac{\vec{\rho}_3}{\rho_3^{\,3}}\right)$$

Note

By using propellant to cancel the effects of lunar gravitation, the problem is reduced to one of restricted three-body motion, and one may hope to keep the spacecraft near an unstable collinear equilibrium point $\mathcal{L}_2$ with very little additional propellant.

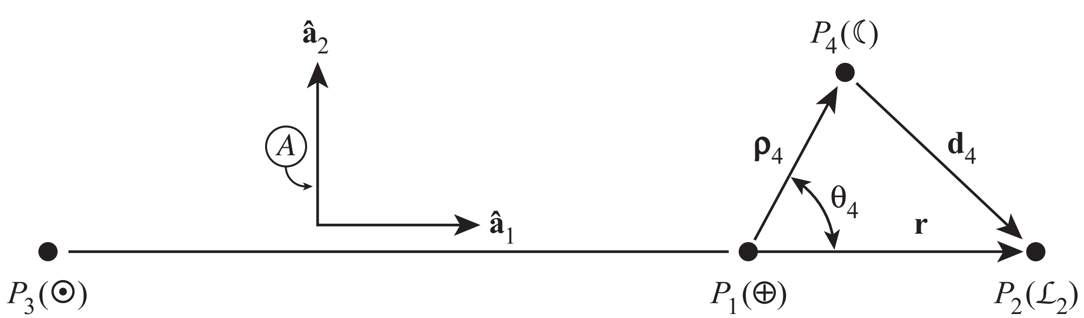
Approximation of Lunar Perturbation

Rewrite the lunar perturbation

$$\frac{\mathbf{p}}{m_2} = Gm_4\left(\frac{\mathbf{d}_4}{d_4^{\,3}} + \frac{\vec{\rho}_4}{\rho_4^{\,3}}\right) = Gm_4\left(\frac{\mathbf{r} - \vec{\rho}_4}{\left|\mathbf{r} - \vec{\rho}_4\right|^3} + \frac{\vec{\rho}_4}{\rho_4^{\,3}}\right)$$

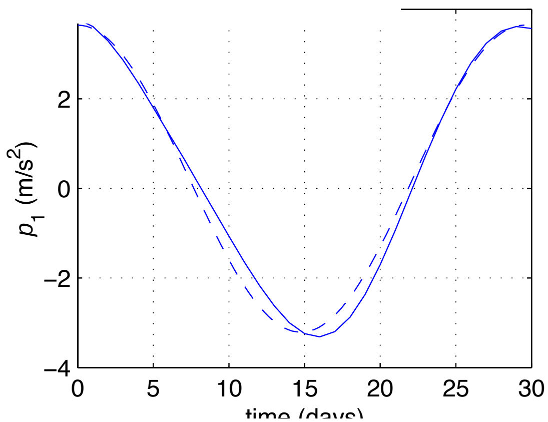
Use binomial expansion ($r \approx 4\rho_4$) and neglect inclination of moon's orbit plane to the ecliptic,

$$\frac{\mathbf{p}}{m_2} \approx Gm_4\left\{\left[\left(\frac{1}{\rho_4^{\,3}} + \frac{2}{r^3}\right)\rho_4\cos\theta_4 + \frac{1}{r^2}\right]\hat{\mathbf{a}}_1 + \left(\frac{1}{\rho_4^{\,3}} - \frac{1}{r^3}\right)\rho_4\sin\theta_4\hat{\mathbf{a}}_2\right\}$$

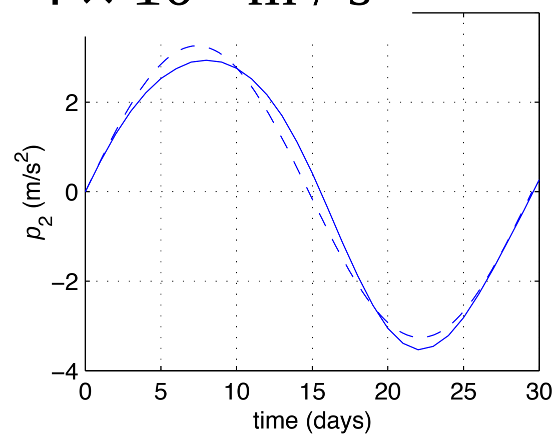


Propulsive force per unit mass to counter lunar perturbation

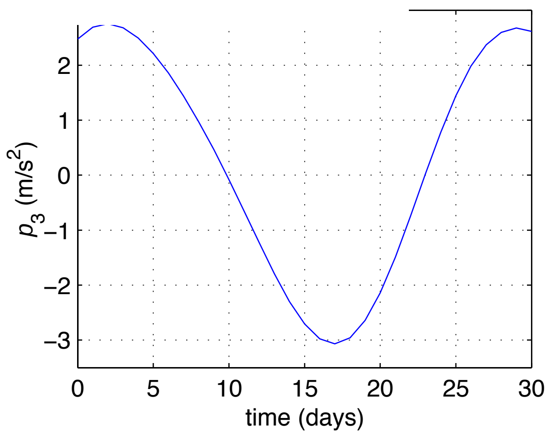
4×10^{-5} m / s^2

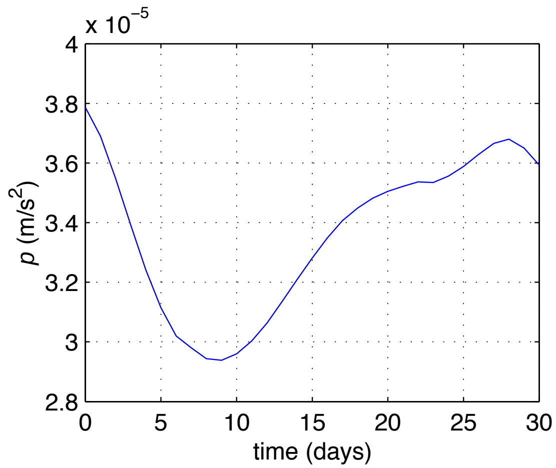


4×10^{-5} m / s^2



3×10^{-6} m / s^2





Exact, using ephemerides for Earth and moon

Approximate, using

$\rho_4 = 384{,}400$ km

$r = 1.50151 \times 10^6$ km

$Gm_4 = 4.903 \times 10^3$ km^3/s^2

$Gm_1 = 3.986 \times 10^5$ km^3/s^2

Estimate of ΔV

Integrate the approximate expression for **p**/m_2

$$\Delta V_1 = \int \frac{|\mathbf{p}\cdot\hat{\mathbf{a}}_1|}{m_2}\,dt \approx 4m_4\sqrt{\frac{G}{m_1\rho_4}}\left[1+\frac{\pi}{2}\left(\frac{\rho_4}{r}\right)^2+2\left(\frac{\rho_4}{r}\right)^3\right]$$

= 57 m/s per month

$$\Delta V_2 = \int \frac{|\mathbf{p}\cdot\hat{\mathbf{a}}_2|}{m_2}\,dt \approx 4m_4\sqrt{\frac{G}{m_1\rho_4}}\left[1-\left(\frac{\rho_4}{r}\right)^3\right]$$

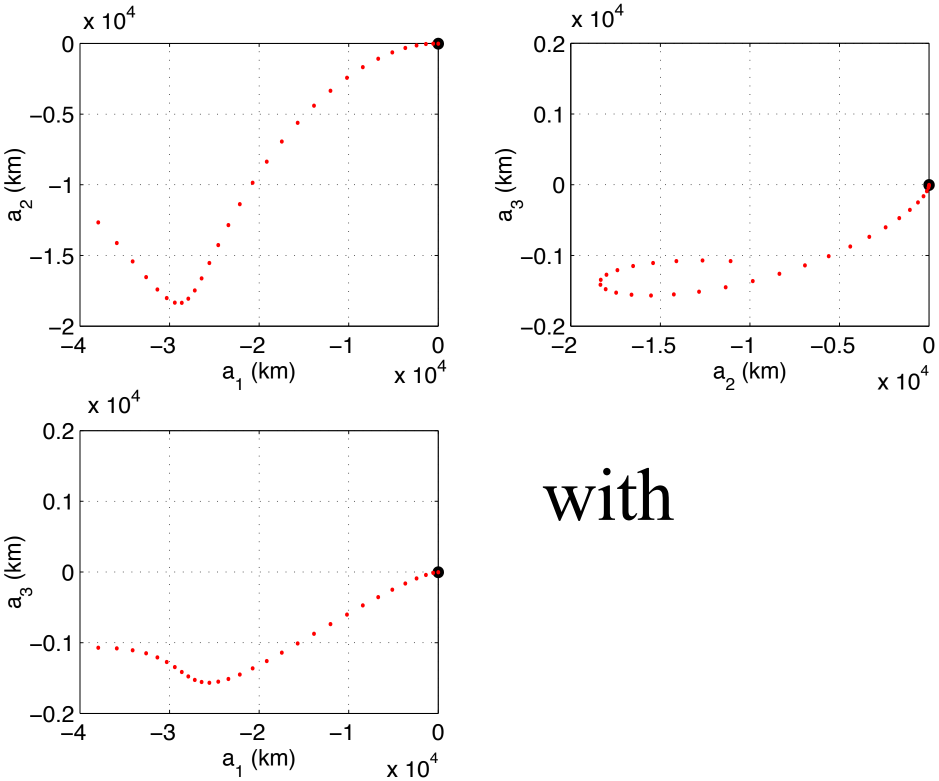
= 49 m/s per month

Total ΔV per month:

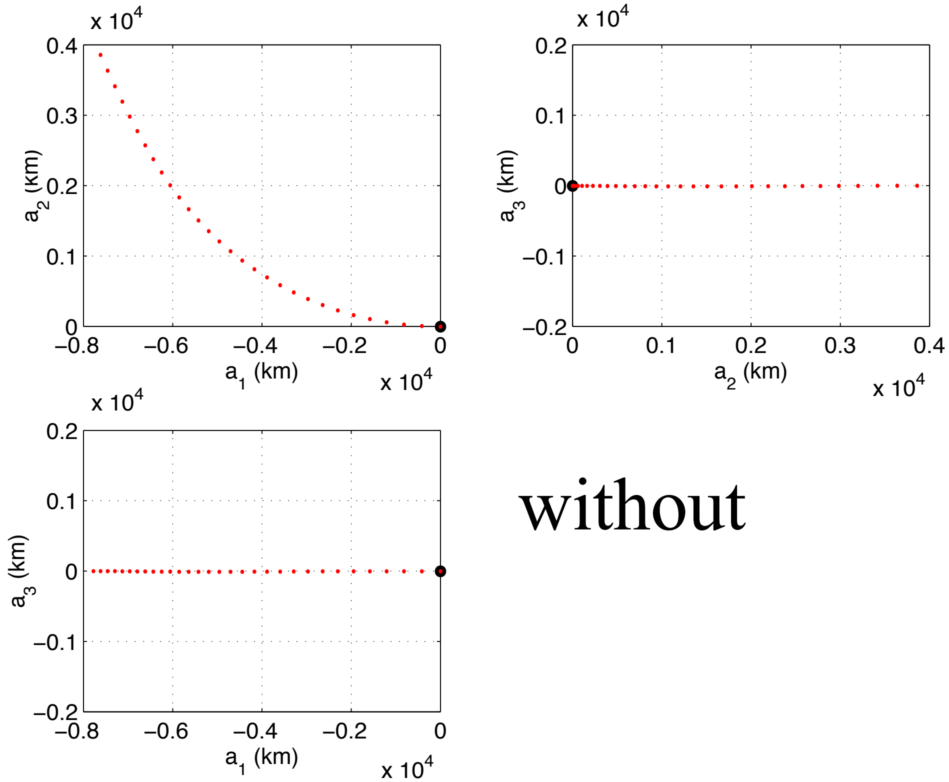
106 m/s to hold S/C fixed, coincident with $\mathcal{L}_2$

49 m/s to allow S/C to move along the Sun-Earth line

Excursions with and without lunar perturbation



with



without

Conclusion

- S/C must have propulsion to counter lunar perturbation.
- Lunar perturbation is expressed analytically, and evaluated numerically.
- Analytic and numerical estimates given for ΔV.
- Allowing S/C to move along Sun-Earth line requires less than half the ΔV needed to keep it fixed at $\mathcal{L}_2$.
- First order analysis provided here: results obtained with optimal control presented in next paper (interesting motion!).